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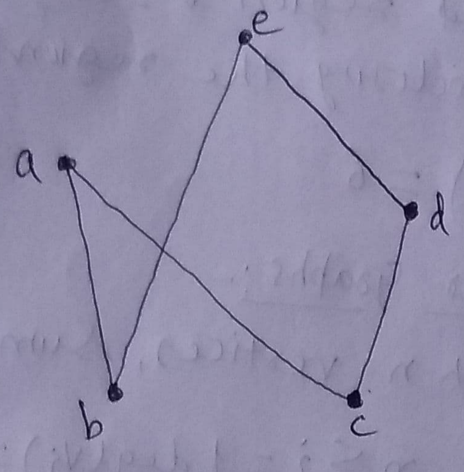
M.Sc. Sem-II, Paper-V

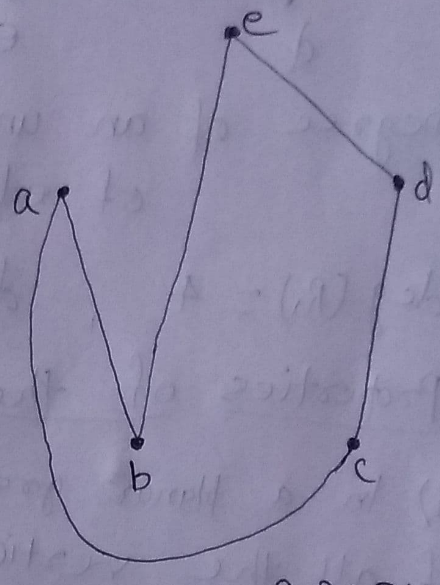
Graph Theory (Planar Graph and Their Properties)

A graph G is said to be planar if it can be drawn on a plane or a sphere so that no two edges cross each other at a non-vertex point.

Example



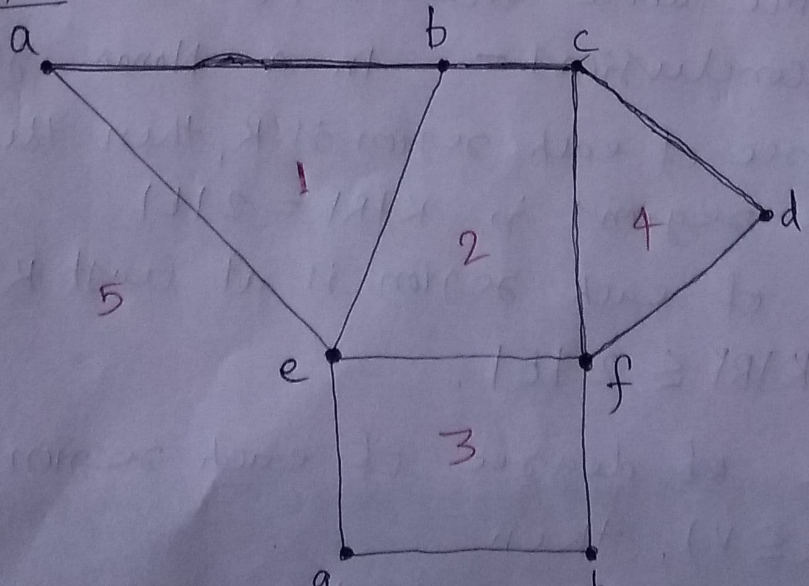
Non-PLANAR GRAPH



PLANAR GRAPH

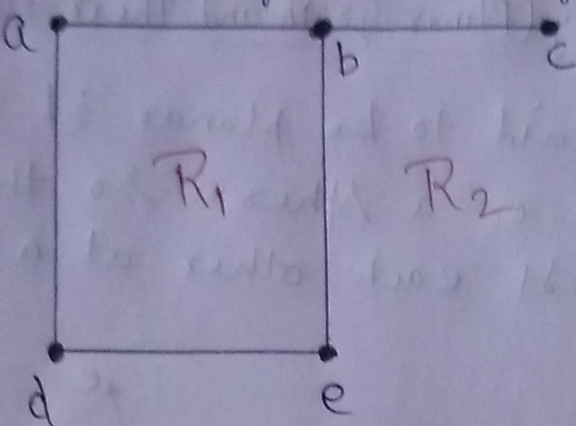
Regions: Every planar graph divides the plane into connected areas called regions.

Example



Degree of a bounded region r = deg(r) = Number of edges enclosing the regions r.

deg (1) = 3 , deg (2) = 4 , deg (3) = 4 , deg (4) = 3 , deg (5) = 8



Degree of an unbounded region r = deg(r) = Number of edges enclosing the regions r.

deg (R_1) = 4 , deg (R_2) = 6

Properties of the planar Graphs :-

1) In a planar graph with n vertices, Sum of degrees of all the vertices is $\sum_{i=1}^{n} deg(v_i) = 2|E|$.

2) According to Sum of Degrees of Regions theorem, in a planar graph with n regions, Sum of degrees of regions is $\sum_{i=1}^{n} deg(r_i) = 2|E|$.

Based on the above theorem, we can draw the following conclusions - In a planar graph,

- If degree of each region is K, then the sum of degree of regions is $K|R| = 2|E|$
- If the of each region is at least K ($\geq K$), then $K|R| \leq 2|E|$.
- If the ~~of~~ degree of each region is at most K ($\leq K$), then

$K|R| \geq 2|E|$

Note: Assume that all the regions have same degree

3) According to Euler's formulae on planar graphs,

* If a graph G is a connected planar, then

$$|V| + |R| = |E| + 2$$

* If a planar graph with K components, then

$$|V| + |R| = |E| + (K+1)$$

Where, $|V|$ is the number of vertices, $|E|$ is the number of edges, and $|R|$ is the number of regions.

4) Edge vertex Inequality :- If G is a connected planar graph with degree of each region at least K, then $|E| \leq K/K-2 \{|V|-2\}$

We know, $|V| + |R| = |E| + 2$

$$K|R| \leq 2|E|$$

$$K(|E| - |V| + 2) \leq 2|E|$$

$$(K-2)|E| \leq K(|V|-2)$$

$$|E| \leq \frac{K}{K-2}\{|V|-2\}$$

5) If G is a simple connected planar graph, then

$$|E| \leq 3|V| - 6$$

$$|R| \leq 2|V| - 4$$

There exists at least on vertex V ~~in~~ G, such that $\deg(V) \leq 5$.

6) If G is a simple connected planar graph (with at least 2 edges) and no triangles, then

$$|E| \leq \{2|V| - 4\}.$$

7) Kuratowski's theorem :- A graph G is non-planar if and only if G has a subgraph which is homeomorphic to K_5 or $K_{3,3}$.